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STUDY AND DESIGN THE HYDRODYNAMIC DEEP DRAWING PROCESS FOR BLANKS OF NON UNIFORM THICKNESS

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ABSTRACT:- In this study a numerical procedure was proposed for the design of deep drawing process using finite element method (F.E.M) through program code (ANSYS 11) simplified 2-D ax symmetric model of conical cup are been developed. The process of hydrodynamic deep-drawing (HDD) has been modified to draw tapered blanks of small angles , this option unattainable in classical deep-drawing processes has its applications in producing specially –dedicated products .But more generally this process is aimed at decreasing the demand for a strict uniformity of blank thickness and thus relaxing the costly tight tolerances on the preformed blanks. The idea rests on changing the customary clamped die to a self aligned die by letting it be semi- spherical with rotational degrees of freedom.

 The automatic self alignment of the die to the current angle of the blank arises from the very nature of the hydrodynamic fluid pressure beneath the blank.

 The solution for the statically admissible stresses throughout the flange is produced in an asymptotic expansion fashion. With the inclusion of thickness variation interfacial friction and exponential hardening of the material. Expressions for the limit drawing ratio (LDR) as a function of the angle of the tapered blank (beside other relevant parameters) are developed . They are compared favorably with experiments produced on a specially built apparatus.

NOTATION

- *A*a function defined in equation (6)
- *a* inner radius of the flange
- *b* outer radius of the blank

- *c* material constant defined in equation (20)
- *F* punch load
- *g* the gap (clearance height) between the flange and the die
- *k* yield in shear
- *m* frictional shear factor
- *n* strain hardening coefficient
- *p* fluid pressure distribution under the flange
- *pa* hydrostatic pressure in the container beneath the cup
- *R* current radius of the flange
- *R'* radius of the flange at rupture (position of maximum punch load)
- t thickness (a variable)
- t_0 maximum thickness (a constant)
- \mathbf{u}_0 punch speed
- w a function defined in equation (12)
- α_{0} the angle of the tapered thickness
- η fluid viscosity
- σ_{ii} components of stress
- ε equivalent strain
- μ Coulomb coefficient of friction

INTRODUCTION

Production of cups with intentionally non –uniform thickness is relatively rare. Some specific uses of such products known are in military applications. But unintentionally such circumstances are frequent. In fact every blank has some degree of non – uniformity. the drawing process can normally be terminated without interruptions. If however the blank is non _ uniform in its thickness ,the oversized portion may reach an impasse and the process will be chocked. Any relaxation (or better _ prevention) of this drawback should be highly beneficial. One method of doing so is to use the hydroforming process $(1-3)$. In this process the blank is counteracted against a pressurized fluid and therefore any variation in the thickness of the blank is tolerable . the process itself demands sophistication in controlling the fluid pressure and a considerable enhancement of the capacity of the loading system

compared to conventional processes . In the alternative suggestion is based on endowing a rotational degree of freedom to the otherwise clamped die in the new hydrodynamic deep _ drawing (HDD) process illustrated in figure (1). This process has its own disadvantages in regard to slow operation and delicacy in the initial construction of the machine ⁽⁴⁾. the fact that the fluid flows beneath the blank (rather than acting hydrostatically as in the hydroforming process) enables the die to adjust itself continuously and stably (5) to the variations in the blank thickness .

The capability of this process to produce cups with non _ uniform thickness is de monstrated using a specially _ built machine. The highly non _ linear equilibrium equations, along with yielding of exponential hardening material , are solved using perturbation analysis. The first two terms _ the zero and the first order solution _ were used in this paper to anticipate rupture site (with surprisingly counter – intuitive results) and LDR. Some similarities to the drawing force behavior during its travel , indicated by analytical (6) , Research and development in sheet metal forming processes requires lengthy and expensive prototype testing and experimentation in arriving at a competitive product. The overall quality and performance of the object formed depends on the distribution of strains in the sheet material. Material properties, geometry parameters, machine parameters and process parameters affect the accurate response of the sheet material to mechanical forming of the component. The stretching primarily depends on the limit strains. The limit strains are sensitive to strain distribution $(7,8)$. Numerical $(8,9)$ or experimental $(10,11)$ studies of classical drawing processes , Here we focus mainly on reaching LDR using experiments (with especially _ rupture behavior by an asymptotic _ expansion stress analysis.

FORMULATION

 Consider a tapered blank , as show in fig .(1 b). The blank has an initial radius of b and thickness t_o . the non Δ uniformity in the thickness is measured by the angle α_o .

 A cup of radius *a* is formed by the downward travel , *h* , of the punch . The maximum depth of the cup depends , of course , on the maximum permissible drawing ration $(1/(\frac{b}{a})$ max (called LDR), beyond which rupture intervenes. If the blank is additionally tapered (by α _o), the LDR becomes also, as will be shown, a function of this angle. This relationship is going to be found by using (non \equiv linear and non – steady) stress analysis , utilizing the perturbation method. The asymptotic expansion (with the small parameter α_o)

of the unknown stresses brings to light the effect of the tapered thickness on the punch load , on rupture location and on LDR. The governing equation are :

(a) the equilibrium equations:

$$
\frac{r\partial\sigma_r}{\partial_r} + \sigma_r \frac{r}{t} \sin\alpha + \tau_r \frac{r}{t} (1 + \cos\alpha) + \sigma_z \frac{r}{t} (\sin\alpha\cos\alpha) = \sigma_\theta - \sigma_r \tag{1}
$$

$$
\frac{\partial \sigma_{\theta} r}{\partial \theta t} + \sigma_{\theta} \sin \beta = \sigma_z \cos \alpha \sin \beta \cos \beta \tag{2}
$$

Where $\alpha = \sin^{-1} (\alpha_o \cos \theta)$

$$
\beta = \sin^{-1} (\alpha_o \frac{1 - \cos \theta}{\theta})
$$

$$
t = t_o - \alpha_o (b - r_o \cos \theta)
$$

$$
r_o = \sqrt{h(1 + 2a) + r^2}
$$
 (3)

(b) the T resca yielding criterion (or von Mises criterion, provided that the normal stress σ , is relatively small):

$$
\sigma_{\text{max}} - \sigma_{\text{min}} = \sigma_r - \sigma_\theta = 2k \tag{4a}
$$

Or for hardening material

$$
\sigma_{\text{max}} - \sigma_{\text{min}} = \sigma_r - \sigma_\theta = C \varepsilon^n \tag{4b}
$$

Where $\sigma = C\varepsilon^{n}$ represents the material behaviour with C and n as material properties. The normal stress distribution, σ_z (denoted hereafter as p), is equivalent to (and a result of) the flowing fluid beneath the blank . This hydrodynamic pressure distribution is calculable beforehand, as shown in [4] , and is shown to depend on the fluid viscosity η , the gap g (between the die and the blank) through which the fluid flows and the punch speed u_0 according to

$$
p(r) = A(...) \lim_{r} \left(\frac{R}{r}\right) \tag{5}
$$

Where , as in (4) but with more algebra,

$$
A(...)=6\eta \frac{u_o}{g^2} \left[\frac{(a+t_o-b\alpha_o)^2}{g} + \frac{t_o^2 - 2t_o b\alpha_o - 2ab\alpha_o + 2at_o}{2(t_o - b\alpha_o)} \right]
$$
(6)

And R is the current radius of the flange.

 The interfacial shear between the blank and the blank holder is prescribed by the normal fluid pressure (5) and is given by

$$
\tau_r = \mu \, p(r) \tag{7}
$$

The rest of the unknowns, σ_r and σ_θ are solved using equations (1) and (2), along with the traction _ free boundary condition at the far edge of the rim , namely

$$
\sigma_r(R) = 0\tag{8}
$$

SOLUTION BY PERTURBATION WITH ASMALL PARAMETER

Consider the case where non _ uniformity of the blank thickness is relatively small say by an angle α_o *of* 1^0 or less. One can in this case expand the radial stress , σ_r , in terms of this small parameter as

$$
\sigma_r = \sigma_r^0 + \alpha_o \sigma_r^1 + \alpha_o^2 \sigma_r^2 + \alpha_o^3 \sigma_r^3 + \dots
$$
\n(9)

 By substituting the two leading terms of equation (9) into (1) and collecting terms of the same order of magnitude , the following two equations result :

$$
r\frac{\delta\sigma_r^{\ \rho}}{\partial_r} + \frac{\mu A_r (1 + 1/w) \ln(R/r)}{t} + 2k = 0
$$
\n(10)

$$
r\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r^0}{t} \cos \theta + \frac{r}{tw} A(\ldots) \ln(\frac{R}{r}) \cos \theta = 0
$$
 (11)

Where , for abbreviation ,

$$
w = \left(\frac{1}{1 - \alpha^2}\right)^2 - \alpha_o^2 \sin^2 \theta\right)^{0.5}
$$
 (12)

The solution of equation (10) with

 $\sigma_r = 0$ *at* $r = R$

$$
\sigma_r^0 = 2k \ln(\frac{R}{r}) + \frac{1}{2} \mu A(\dots)(1 + \frac{1}{w})\frac{1}{t}[R - r\{\ln(\frac{R}{r}) + 1\}]
$$
\n(13)

By substituting equation (13) into (11) and integration with respect to

while using the boundary condition (8) , one gets σ_r

$$
\sigma_r = \sigma^0 \circ \frac{r \cos \theta}{t} + \alpha \circ A(\dots) \frac{\cos \theta}{2wt} [r^2 \ln(\frac{R}{r}) + \frac{1}{2} (r^2 - R)] \tag{14}
$$

In the case of a material with power _ low hardening, the constant yield stress, 2k, in equation (10) is replaced by the inhomogeneous yield stress readily shown (in [6] to yield

$$
2k = C3^{n/2} \frac{(b^2 - R^2)^n}{r^{2n}} \tag{15}
$$

The results of resolving equations (10) and (11) are hence

$$
\sigma^o_r = \frac{C(b^2 - R^2)^n}{2n\sqrt{3^n}} \left(\frac{1}{r^{2n}} - \frac{1}{R^{2n}}\right) + \frac{1}{2}\mu A(\ldots)(1 + \frac{1}{w})\frac{1}{t}[R - r\{\ln(\frac{R}{r}) + 1\}]
$$
(16)

$$
\sigma_r = \sigma^0 \left[\frac{r \cos \theta}{t} + \alpha_o A(\ldots) \frac{\cos \theta}{2wt} \left[r^2 \ln \frac{R}{r} + \frac{1}{2} (r^2 - R^2) \right] \right]
$$
(17)

A representative stress distribution based on equations (16) and (17)

The associated hoop stress is obtained from equation (4b). Upon letting the tapered blank approach a uniform blank (i.e. setting α _o to zero), equation (16) is reduced to the solution already known⁽⁴⁾.

THE LIMIT DRAWING RATIO (LDR)

The stress distribution is enhanced by increasing the drawing ratio. In the limit, the drawing process is interrupted by either buckling of the flange, or rupture of the cup (see Fig. 1b) Buckling is caused by excessive compressive hoop stress; rupture is due to excessive radial tensile stress, which leads to plastic instability. The limit drawing ratio (LDR) is reached when the process is terminated just before one of the failure modes intervenes .The buckling phenomenon is known to be highly sensitive to the thickness (2) . If it occurs, it is expected to start at the thinnest portion of the blank, as indeed happens in reality. No analysis

is yet available to forecast its appearance in HDD processes. The limiting factor in the present study was the failure by rupture (identified as the point where the tensile stress reaches its maximum permissible value The necking which precedes the rupture was situated , in most cases , in the thickest portion of the cup wall , near the lip (see fig . 1b.) .

At the lip , the circumferential dimension of the cup remains unchanged so that a plane strain condition prevails . In this case , the maximum permitted strain before necking occurs is equal to the strain _ hardening exponent :

$$
\varepsilon_{\text{max}} = n \tag{18}
$$

Consequently , the stress along the wall of the cup cannot exceed $\sigma_{\text{max}} = C \varepsilon^n \max_{\text{max}} \sigma_{\text{max}}$ (19)

Which marks the rupture inception of the cup .

For the non _ hardening case , the stress criterion of

$$
\sigma_{\text{max}} = 2k \tag{20}
$$

Is used to predict rupture.

Let us now increase the drawing ratio, b/a , to the limit where the radial stress thus generated [say by equation (13), for simplicity] reaches its critical value [equation (20)]. This extreme value, (b/a) , which designates the LDR, can now be solved. Set $r = a$, $R = b$ and in equation (13) (in order to furnish the most severe radial stress) and equate it to (20) . The result is

$$
\sigma^0_{r(\text{max})} + \alpha_o \sigma_{r(\text{max})} = 2k \tag{21}
$$

For the hardening case, one sets $r = a$, and and replaces R by (R^*) in equations (16) and (17). (R^*) is the flange radius when rupture occurs. This condition is predicted by searching for the maximum radial stress , expressed in equations (16) and (17) , with respect to R . It reads approximately

$$
\frac{R^*}{a} = \left(\frac{b}{a}\right)_{\text{max}}^{1/(1+n)}
$$
(22)

The LDR is thus solved implicitly from

$$
\sigma^0_{r(\text{max})} + \alpha_o \sigma_{r(\text{max})} = C n^n \tag{23}
$$

FINITE ELEMENT ANALYSIS

 Using ANSYS-11 the finite –element analysis was simulated by building a model same as the assumed model in theoretical analysis figure (1) .having environmental effect similar to that of reality and theoretical assumption .The model has to be meshed to a specific shape of element according to the chosen element which is solid (plane-82)with 8 node element defined by eight nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element (preferred) or as an ax symmetric element. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. Because of symmetry (about x-axis) one half of the model was first solved later the results showed by symmetry expansion of results. The loading process in ANSYS-11 is employed by applying a horizontal displacement to a work piece at time step according to the flow velocity at different load step option. The local coordinate technique was used to locally orient the axis of movement of the work piece to change with the curved pass. The obtained results after applying solves command were stress, strain shear force, shear stress and energy in both x and y coordinates.

Coulomb friction was assumed in all contact interfaces with the friction coefficient (μ) $= 0.1$). The die and the blank holder were completely constrained. The punch, which was slightly tilted in relation to the die (tilt angle 0.4 degree), was prescribed a displacement in vertical direction and was free to move in the horizontal direction. Most of the Contact (48 2D) Node -to-Surface Contact Element are penalty based. With a penalty based contact algorithm it is impossible to avoid node penetration if there is contact as shown in figure (2 ,3).

The solution was done by loading the model in two steps:

- 1. constraining the die by exerting a displacement equal to zero to all die surfaces
- 2. and making the time stepping control off .

Loading the model with constant pressure changed from 300 to 1200 Mpa in a separate loading, and exert a sliding distance to Billet surfaces in the x-direction in each time step which depends on the velocity of exerting the extrusion pressure (5 mm per minute) in each time step the pressure is constant and the same areas loaded with x-axis displacement to analogue with the Billet moving when it is compressed be exerting a compression pressure. When the Billet material flow from the other end of the model which was hollow with a 3mm diameter whole, the extrusion process is

done. Average von misses stress, strain and contact pressure was recorded (although stress, strain and contact pressure can be obtained with different axis). Results estimating drawing pressure with average stress.

RESULTS AND DISCUSSION

Tests were conducted with aluminum _ killed _ steel blanks (SAE 1012).The thickness of the thick side of the blank remained always $t_o = 1$ mm (for common reference) but there were various degrees of tapered thickness (α _o) = 0.25, 0.50, 0.75 and 1.0). The LDR was reached by increasing gradually the ratio (b/a) of the drawn blanks until rupture intervened . The results were recorded in Fig .4. and show satisfactory agreement with the computation based on equation (21) and (23) .

The peculiar fact that ruptures starts at the thickest portion of the blank (rather than at the thinnest) is explained by the stress solution shown in Figure(3). The severe decrease of LDR with increase in the degree of non \angle uniformity of the blank thickness is demonstrated, indicating the limitation in manufacturing cups with non _ uniform thickness. (This result is not necessarily inherent to this HDD process is its ability to draw cups with non _ circular walls, like polygonal shapes⁽⁴⁾. Therefore, the drawing of blanks having, intentionally or occasionally, a slightly tapered cross section into non _ circular shapes seems now an admissible goal.

At present, the process is relatively slow and not suited to mass production. It seems. However, that with automation of the charge / discharge procedures, the turn around time can be shortened to a degree where industrial applications might be considered.

 Figure (4) The simulation result provides large stress values at the center of the cup, because it does not have bending resistance at the punch shoulder .The effect of punch stroke on the effective stress and effective strain distribution over the cup wall more uniform distribution the more reasonable values of stress are for the value of friction $= 0.1$. The thickness distributions shown in Figure (5) could indicate that the punch initially had been displaced slightly off centre and that during the deep drawing and ironing, the punch had been pushed in the opposite direction. According to the craftsmen on the shop floor it was sometimes possible to improve the can height distribution by slightly offsetting the punch. It can thus be concluded that it may be possible to produce a can with an even height but without having an even wall thickness and it is thus questionable if evenness of the cup

height can be used as a quality measure. This because, in the opinion of the author, it is more important for the subsequent ironing stages to have an even can wall thickness in the circumferential direction than to have an even can height.

Figure (6) show the contours value of plastic stress equivalent with min value $=0.141E+10$ and max value $=0.156$ E+10 . Figure (7) shows the contours Vonmises stress sheet with min value = $0.122E+10$ and max value = $0.152E+10$

fig.(8). Shows X-axis stress distribution is a most important factor affecting drawing process since it result in residual or permanent stress depending on its distribution and amount which is equal to min value $= -0.157E+10$ and max value $= 0.176E+10$.

It is clear that the radial stress (σ_x) is almost tensile with small value at cup bottom and this stress increases toward the cup wall and becomes approximately zero at cup rim in all directions. Figure (9) show Y component stress with min value $= -0.440E+0.9$ and max value = $0.287E+09$. The nature of the axial stress (σ_y) is generally equal to zero at the cup bottom and changes toward the cup wall in all directions. Figure (10) shows the contours radial strain indicates the thinning effect of the sheet metal during cup formation. Due to the conservation of volume during plastic deformation, where the thickness is the large, the radial is more important. The thickness strain (ϵ_Y) distribution indicates the failure zone in produced cup. It is evident from the figure when r-value decreases the thickness strain increases, and it is seen that the maximum thickness strain is along the direction of 45^{\degree} with respect to RD, since this direction has alow r-value and large thickness. The thickness strain is positive at the cup rim indicating thickening of material. The thinning is more pronounced near the punch profile radius at full draw condition. It is seen from the figure(11) shows the strain (ε _Z) varies with varying the r-value and decreases with decreasing r-value. The effective strain (ϵ_0) is the resultant of these strains. It is seen from the figure the effective strain increase with decrease the r-value and reaches the maximum value at cup rim.

CONCLUSIONS

- 1. The FEM simulations show that when the deep drawing and ironing is carried out with a conventional cylindrical die land, a slight tilt of the die land in relation to the punch can give rise to variations in the can wall thickness and in the cup height.
- 2. The FEM results are in good qualitative agreement with the experimental findings. To improve the robustness of the deep drawing and ironing process it is suggested to

make the die with a circular profiled die land in place of the cylindrical die land, because by making the die land circular profiled, a slight tilt of the die will only give rise to minor changes in the contact conditions in the die land – can wall interface.

- 3. FEM simulations show that the cup quality (cup wall thickness and can height) is nearly unaffected by a slight tilt of the die in relation to the punch when the die is made with a circular profiled die land in place of the cylindrical die land.
- 4. The FEM simulations thus strongly indicate that the deep drawing process becomes significantly more robust when carried out with a circular profiled die land.
- 5. The industrial experience and the FEM simulations show that the deep drawing and ironing process is extremely sensitive to small changes in the geometry of the die land when the die is made with a cylindrical die land. A likely explanation why a die with a cylindrical die land can be made to work just by polishing is that the polishing unintentionally makes the die land slightly profiled.

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Fig.(1 a): Schematic view of the modified hydrodynamic deep drawing process.

Fig.(1 b): Tapered blank with initial radius of b.

Fig.(3): The geometry and die of deep drawing (mesh)**.**

Fig. (4): The effect of punch stroke on effective stress distribution comparison of theory result , F.E.M and Experiment.

Fig.(5): Cup wall thickness as function of the angle and with the distance from the inside bottom as parameter.

Fig.(6): The value of plastic stress equivalent with min value $= 0.141E+10$ and max value $=$ 0.156 E+10.

Fig. (7): Vonmises stress sheet with min value $= 0.122E+10$ and max value $= 0.152E+10$.

Fig. (8): X- component stress with min value = $-0.157E+10$ and max value = $0.176E+10$.

Fig. (9): Y component stress with min value $= -0.440E+09$ and max value $= 0.287E+09$.

Fig.(10): Vonmises elastic strain**.**

Fig.(11): Total strain**.**

دراسة وتصمیم لعملیة السحب العمیق الهیدرولیكي لوعاء غیر متماثل السمك

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الخلاصة

 فـي هـذا البحـث تـم اجـراء تحلـیلات عددیـة اقترحـت لتـصمیم عملیـة الـسحب العمیـق باسـتخدام M.E.F مـن خـلال برنامج .11Ansys بتطبیق نموذج ثنائي الابعاد متماثل حول محور Y للاوعیة المخروطیة.

 عملیة السحب العمیق الهیدرولیكي طورت لرسم الخامة ذات الابعاد المائلة بزاویة صغیرة. وهذا الخیار غیر متاح في عملیة السحب العمیق القدیمة وله اهمیة وتطبیقات في انتاج منتجات ذات اهمیة خاصة. وبصورة عامة تهدف العملیة الـى تقلیـل متطلبـات الانتظـام فـي سـمك الـصفیحة وتقلیـل الـسماحات الـصعبة فـي الانجـاز. الفكـرة مبنیـة علـى تغییـر القالـب المبنـي بـشكل یـسمح بتـصمیمه بـشكل دائـري بـسماحات حریـة دائریـة وخاصـیة التعـدیل الـذاتي للقالـب لزاویـة الـصفیحة تـأتي من الخاصیة الهیدروداینمیكیة لضغط السائل تحت الصفیحة.

 الحل للاجهاد الستاتیكي وجد من خلال استخدام خاصیة التماثل حول محـور مـع وجـود الاخـتلاف بالـسمك وتـأثیر الاحتكاك والصلادة للمادة, ومتغیرات لنسبة السحب كدالة لزاویة الـصفیحة المائلـة بالإضـافة إلـى العوامـل الاخـرى. وتقـارن النتائج مع النماذج العملیة التي حصل علیها من أجهزة مختبریة.