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ESTIMATION AND PLOT OF ELECTRICAL FIELD USING FINITE DIFFERENCE METHOD

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ABSTRACT: - Calculation of electric fields with the aid of an computer is now an inevitable tool in various electricity-concerned technology, in particular, for analyzing discharge phenomenon and designing high voltage equipments .The calculation of electric fields generally require higher accuracy, because the highest electric field stress on insulator is usually the most important and decisive value in insulation design or discharge study. This is one of reason why the boundary-dividing methods are preferred to the region-dividing ones, such as finite difference method (FDM) or finite element method (FEM).

The finite difference method is a powerful numerical method for solving partial differential equations. An FDM method divides the solution domain into finite discrete points and replaces the partial differential equations with a set of difference equations. Thus the solutions obtained by FDM are not exact but approximate. However, if the discretization is made very fine, the error in the solution can be minimized to an acceptable level.

In this research grid of finite difference method divided (N by N), N represents number of nodes. In our calculation we take many cases, each case contains specific number of nodes such (7, 15, 25). Then we estimate electric field for different charges values and their locations. We depend on equation (AX = b) .Where A matrix represents node values (depend on boundary condition and operating nodes), X matrix represent electric potential, b matrix represents charges values. X estimation using gauss sideral method and successive over relaxation method .Then we calculate residual which calculated by equation (residual = b - AX). Then we estimated and plot Vx , and Vy. We approve accuracy of our calculation by less quantity of residual, which mean X reach to exact solution.

Also we approve the residual value increased with number of nodes increase because we need to more calculations also the distance between charges increase.

1. INTRODUCTION

To help visualize how a charge, or a collection of charges, influences the region around it, the concept of an electric field is used. The electric field E is analogous to g, which we called the acceleration due to gravity but which is really the gravitational field. The electric field a distance r away from a point charge Q is given by[1]:

$E = k\,Q / r^2$ where k is constant.

- there are two kinds of charge, positive and negative
- like charges repel, unlike charges attract
- positive charge comes from having more protons than electrons; negative charge comes from having more electrons than protons

- charge is quantized, meaning that charge comes in integer multiples of the elementary charge
- charge is conserved

An electric field E can also be defined as the (vectorial) force F that would be exerted on stationary test particle of unit charge by electromagnetic field. A particle of charge q would be subjected to force F=Q.E.

SI Units of electric feild are Newton per Columb ($N.C^{-1}$) , equivalently volt per meter ($V.m^{-1}$) which is terms of SI base units are $kg.m.s^{-3}.A^{-1}$ [2]. An electric field can be visualized on paper by drawing lines of force, which give an indication of both the size and the strength of the field. Lines of force are also called field lines. Field lines start on positive charges and end on negative charges, and the direction of the field line at a point tells you what direction the force experienced by a charge will be if the charge is placed at that point. If the charge is positive, it will experience a force in the same direction as the field; if it is negative the force will be opposite to the field. The fields from isolated, individual charges look like this:

The goals of edge detection are: Produce a line drawing of a scene from an image of that scene, Important features can be extracted from the edges of an image (e.g. corners, lines and curve) and These features are used by higher-level computer vision algorithms (e.g., recognition).Various physical events cause intensity changes:(a.Geometric events such as Object boundary where discontinuity in depth and/or surface color and texture and Surface boundary where discontinuity in surface orientation and/or surface color and texture). (b.Non-geometric events such as Specularity where direct reflection of light, i.e. a mirror, Shadows (from other objects or from the same object and Inter-reflections). The edge detection can be found by four steps:-

1- Smoothing: suppress as much noise as possible, without destroying the true edges.
2- Enhancement: apply a filter to enhance the quality of the edges in the image (sharpening).
3- Detection: determine which edge pixels should be discarded as noise and which should be retained (usually, threshold provides the criterion used for detection).
4- Localization: determine the exact location of an edge (sub-pixel resolution might be required for some applications, that is, estimate the location of an edge to better than the spacing between pixels). Edge thinning and linking are usually required here. The Criteria for good edge filters depend on some factor as following:-

i-No response to flat regions ⇒ Sum of mask values is zero: $\sum(r, c) = 0$.

ii- Isotropy: Response must be independent of edge orientation.

iii- Good detection: Minimize the probabilities of(detecting spurious edges caused by noise and missing real edges)

iv- Good localization: Detected edges must be as close as possible to true edges.v- Single response: Minimize number of false local maxima around true edge [1].

An electric field can be visualized on paper by drawing lines of force, which give an indication of both the size and the strength of the field. Lines of force are also called field lines. Field lines start on positive charges and end on negative charges, and the direction of the field line at a point tells you what direction the force experienced by a charge will be if the charge is placed at that point. If the charge is positive, it will experience a force in the same direction as the field; if it is negative the force will be opposite to the field.

The fields from isolated, individual charges look like figure (1):When there is more than one charge in a region, the electric field lines will not be straight lines; they will curve in response to the different charges. In every case, the field is highest where the field lines are close together, and decreases as the lines get further apart [1].

2. APPROACH

Calculation of electric fields with the aid of a computer is now an inevitable tool in various electricity-concerned technology. In particular, for analyzing discharge phenomenon and designing high voltage equipments.

The calculation of electric fields generally require higher accuracy, because the highest electric field stress on insulator is usually the most important and decisive value in insulation design or discharge study. This is one of reason why the boundary-dividing methods are preferred to the region-dividing ones, such as finite difference method (FDM) or finite element method (FEM). Usually former methods does not need numerical differentiation to obtain field values.

The finite difference method is a powerful numerical method for solving partial differential equations. In applying the method of finite differences a problem is defined by:

• A partial differential equation such as Poisson's equation
• A solution region
• Boundary and/or initial conditions.

An FDM method divides the solution domain into finite discrete points and replaces the partial differential equations with a set of difference equations. Thus the solutions obtained by FDM are not exact but approximate. However, if the discretization is made very fine, the error in the solution can be minimized to an acceptable level.

In applying the methods of finite differences, we define the solution region into a finite number of meshes as shown in Fig (2) [3].

For 2-D case, differential equation simplifies to:-

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial y^2} = f(X,y) \qquad \ldots\ldots..1$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{\Delta x^2} \qquad \ldots\ldots..2$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{\Delta y^2} \qquad \ldots\ldots.3$$

$$\text{Let } \Delta x = \Delta y \qquad \ldots\ldots\ldots 4$$

$$V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1} - 4V_{i,j} = b \qquad \ldots\ldots\ldots 5$$

Where V represents Potential (Volt)

f Represents electrical force (Coulomb) [4].

The calculation of electric field depends on quantity of charges and their locations in x-y plane. Finite difference method (as mention earlier) is more accurate method for this calculation. In this methods, the solution domain is divided into a grid of discrete points or nodes. In this work we estimate many cases. Such different number of nodes (5, 7, 15,25) , also we take different locations of charges at (two ends of grid and middle of grid). If we take number of nodes equal 5 the grid of estimation is shown in figure (3).

For example If there are two charges located at two different points such (2,2) ,(4,4) then we try to find electrical field between these two charges by depend on following steps :-

- The boundaries of our grid lie between $1 \le X \le 5$, $1 \le y \le 5$
- $f(X,y) = \begin{pmatrix} +1 & for \ (X,y) = (2,2) \\ -1 & for \ (X,y) = (4,4) \\ 0 & elsewhere \end{pmatrix}$ where $f(X,y)$ represents electrical force
- Boundary condition is $f(X,y) = 0$ for all boundries do $\Delta X = \Delta y$
- In our grid nodes (1,2,3,4,5,10,15,20,25,24,23,20,21,16,11,6) equal to zero, and electric field on nodes (7,8,9,12,13,14,17,18,19) may be calculated

Where squares represents charges position ((2,2),(4,4)) which equal to (1,-1)

Coulomb that mean red square represents positive charge while blue square represents negative charge , green squares represent boundary points which equal to zero, .
The matrix that represents this case shown in appendix.
We discuss many cases such:-

- Two positive sets of charges located at grid ends. (1,1),(1,2),(1,3),(1,4),(1,5), (5,1),(5,2),(5,3),(5,4),(5,5) which equal to (+1).
- Two sets of different charges located at grid ends (1,1),(1,2),(1,3),(1,4),(1,5) which equal to (-1) and (5,1),(5,2),(5,3),(5,4),(5,5) which equal to (+1).
- Set of negative charges located at middle of grid and two sets of positive charges located at ends of grid or vice versa two negative sets of charges located at ends of grid and set of positive charges located at middle . (1,1),(1,2),(1,3),(1,4),(1,5), (5,1),(5,2),(5,3),(5,4),(5,5) which equal to (+1) or (-1) and (3,1),(3,2),(3,3),(3,4),(3,5) which equal to (-1) or (+1).
- At one end of grid set of positive charges and at other end of grid are set of negative charges and at middle negative or positive set of charges (1,1),(1,2),(1,3), (1,4),(1,5) which equal to (-1) and (5,1),(5,2),(5,3),(5,4),(5,5) which equal to (+1) and(3,1),(3,2),(3,3),(3,4),(3,5) which equal to (-1) or (+1).
- Two positive charges located at ((2,2),(4,4)) ,

We build programs using matlab showing in the figure (4). Main program call three functions one for generating matrix and other to solve equation (AX = b) using Gauss-sideral method and calculate residual value (residual = Ax –b) for different values of N (N represents number of nodes) then plot residuals for all values of N, and there is other function that solve equation (AX = b) using successive over relaxation method for different values of w .

3. RESULT & DISCUSSION

In this part we test many cases that relate to sign of charges and their positions for different number of nodes . Also estimation of X values (when N equal 25 nodes is implemented by two ways gauss-sideral and successive over relaxation methods) .

In first case we consider set of positive charges located at top and bottom of grid. As we notice (from figure (3-1)) the residual value is decrease with increased number of iterations for all values of N (N represents nodes number in grid), there are three different values of N (7, 15 ,25)which indicate to decrease difference between (AX and b)that mean X reach to exact solution. We also notice the residual value increased with increase number of nodes because the distance between nodes increased . In figure (3-2) there is a comparison between two values of w (1.6 ,1.5) that used in successive over relaxtion estimation and gauss-sideral method that used to estimate X values when N equal 25, we notice that w= 1.6 is more accurate than other because residual reach small values than other. In Figure (3-3) we use countour3(a_square,50) function ,where a_sequare is obtained from reshape (X,25,25) function. In this figure we plot electrical potential.

In second case we consider two sets of different charges located at ends of grid. The figures (3-4,3-5,3-6) analogue to figures (3-1,3-2,3-3) in first case. From figure (3-6) we see how different charges attract each other.

In third case we consider set of negative charges located at middle of grid and two sets of positive charges located at ends of grid or vice versa two negative sets of charges located at ends of grid and set of positive charges located at middle of grid. As shown in figures (3-7,3-8,3-9).

In fourth case we consider at one end of grid set of positive charges and at other end of grid are set of negative charges and at middle neagaive or positive set of charges as shown in figures (3-10,3-11,3-12).

In fifth case we consider two charges one positive and other negative located at top and bottom of grid as shown in figures (5,6,7).

From all cases we notice if charges have same charges they repel each other and they attract if have different charges.

4- Conclusion

From figures (8, 9, 10, 11, 12) we conclude that the increase number of nodes leads to less accuracy because the calculation increase, but in all cases the accuracy is acceptable. From figures (13, 14, 15,16, 6) we notice the successive over relaxation is more accurate than gauss-sideral method in such application especially when w = 1.6 , this method such modified version from gauss-sideral method ,the value of w is varied between 1 and 2, so the designer select the best value of w.

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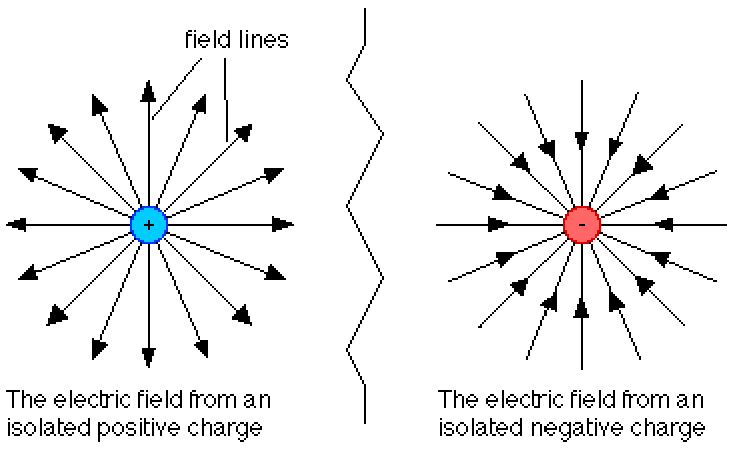

Figure (1): fields from isolated, individual charges

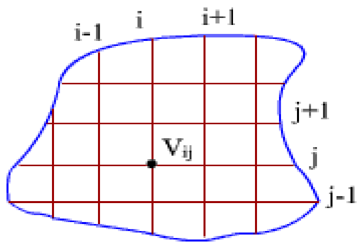

Figure (2): finite number of meshes

21	22	23	24	25
16	17	18	19	20
11	12	13	14	15
6	7	8	9	10
1	2	3	4	5

5,1	5,2	5,3	5,4	5,5
4,1	4,2	4,3	4,4	4,5
3,1	3,2	3,3	3,4	3,5
2,1	2,2	2,3	2,4	2,5
1,1	1,2	1,3	1,4	1,5

Figure (3): number of nodes equal 5 the grid of estimation

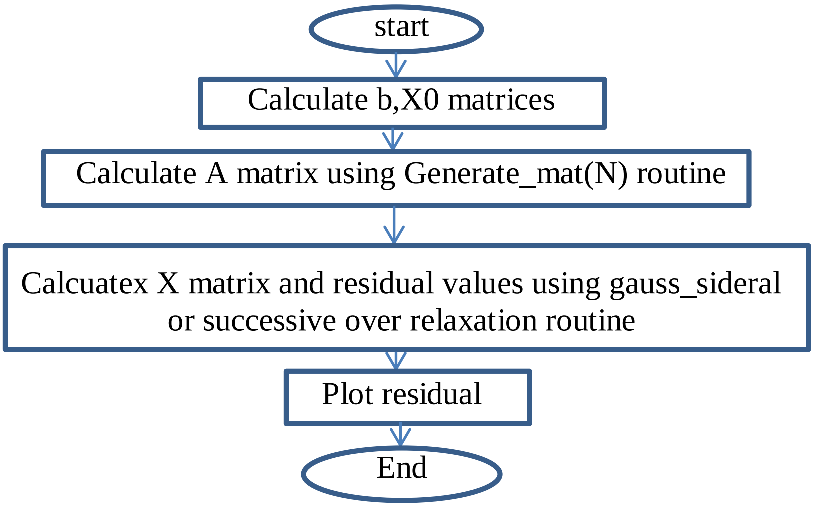

Figure (4): programs algorithm

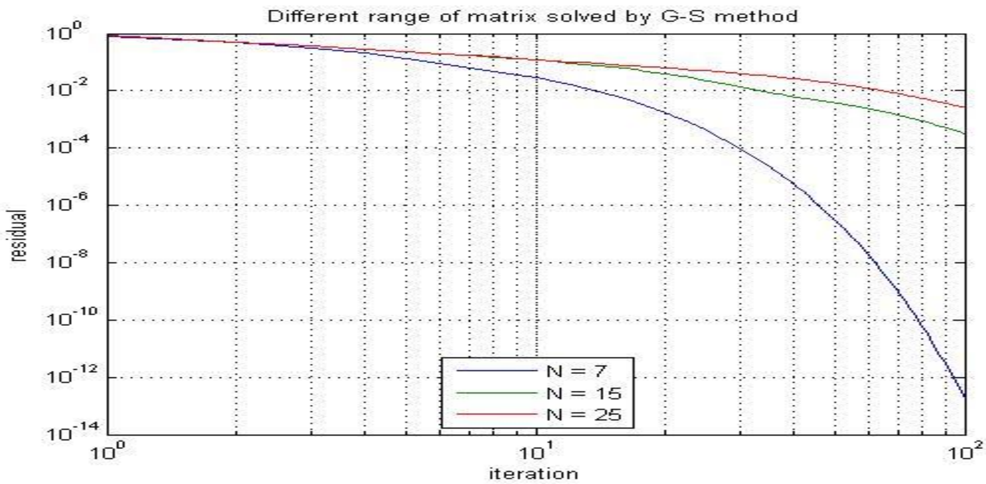

Figure (5): Two charges one positive and other negative located at top and bottom of grid

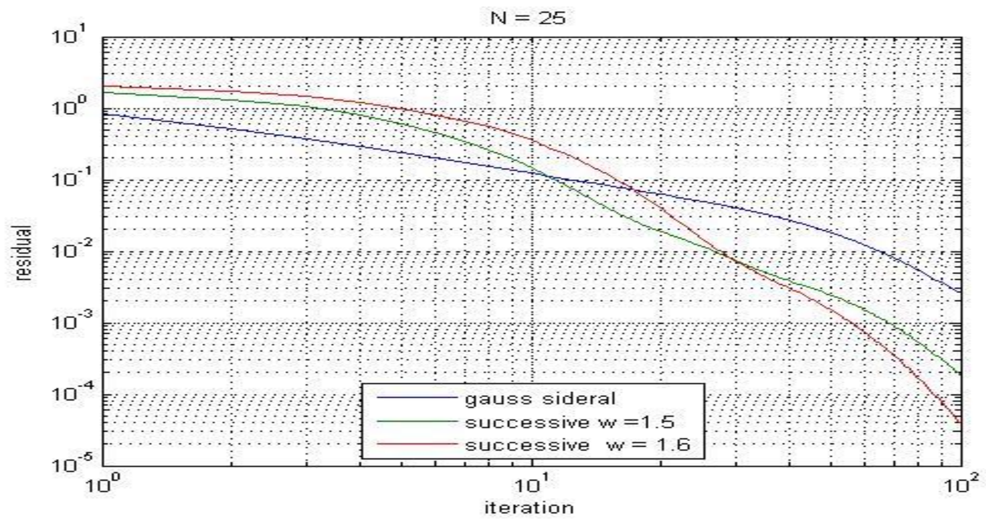

Figure (6): Two charges one positive and other negative located at top and bottom of grid

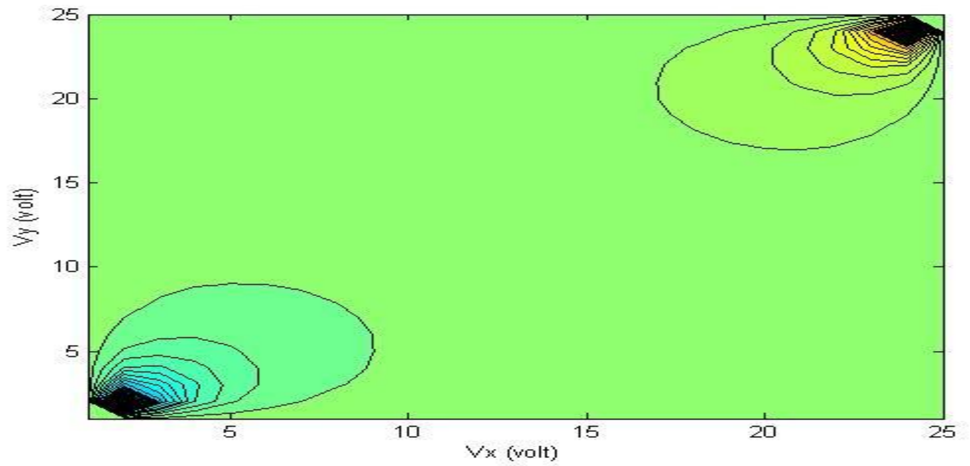

Figure (7): Two charges one positive and other negative located at top and bottom of grid

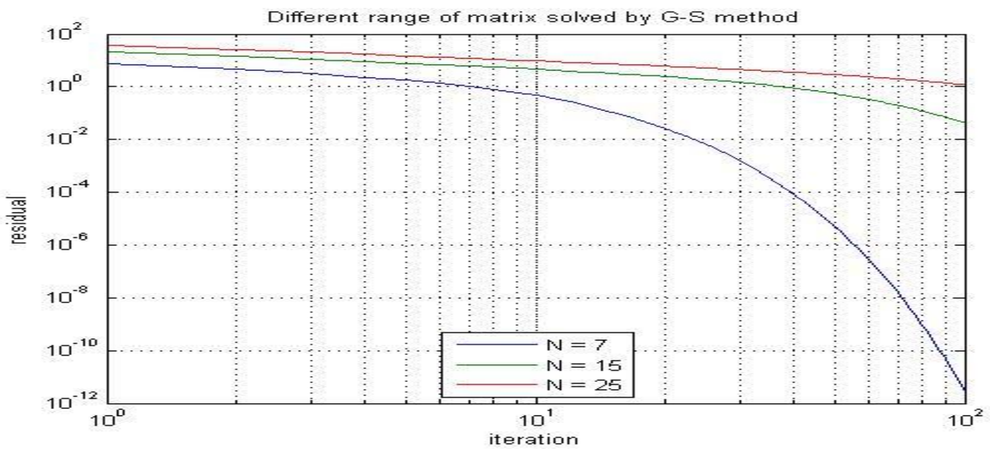

Figure (8): Two positive sets of charges located at grid ends

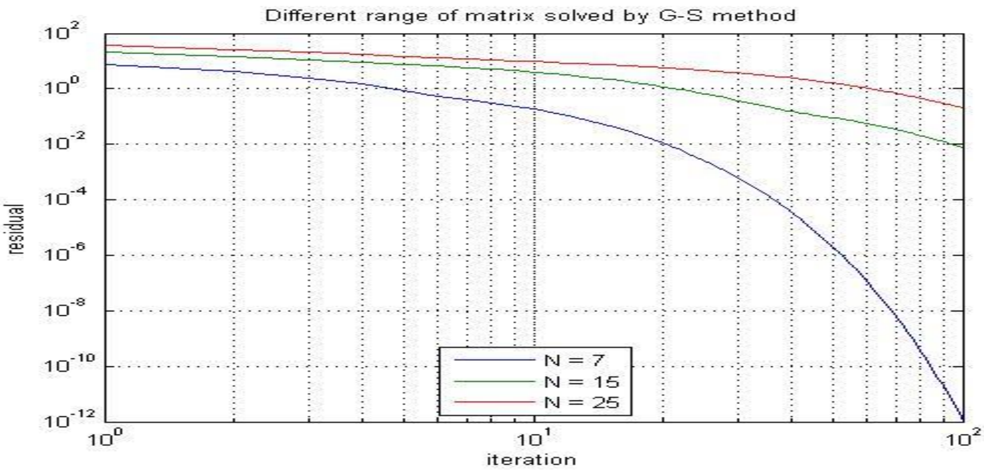

Figure (9): Two sets of different charges located at ends of grid.

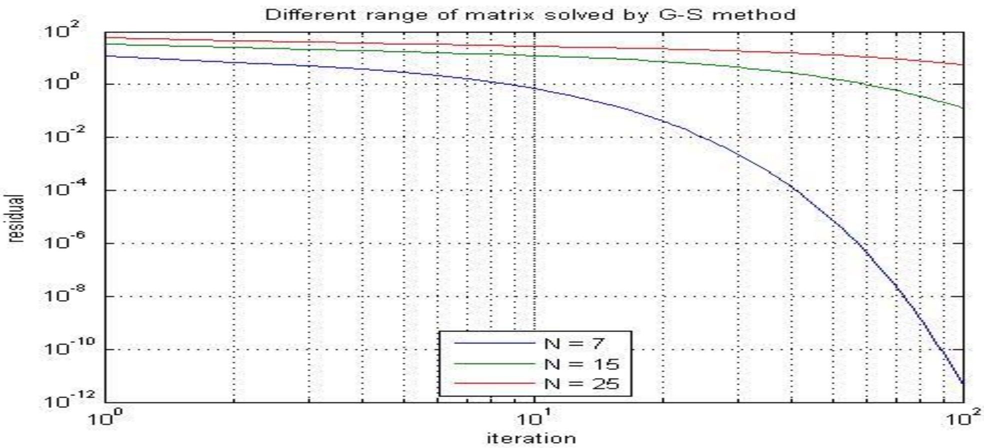

Figure (10): Set of negative charges located at middle of grid and two sets of positive charges located at ends of grid or vice versa two negative sets of charges located at ends of grid and set of positive charges located at middle of grid

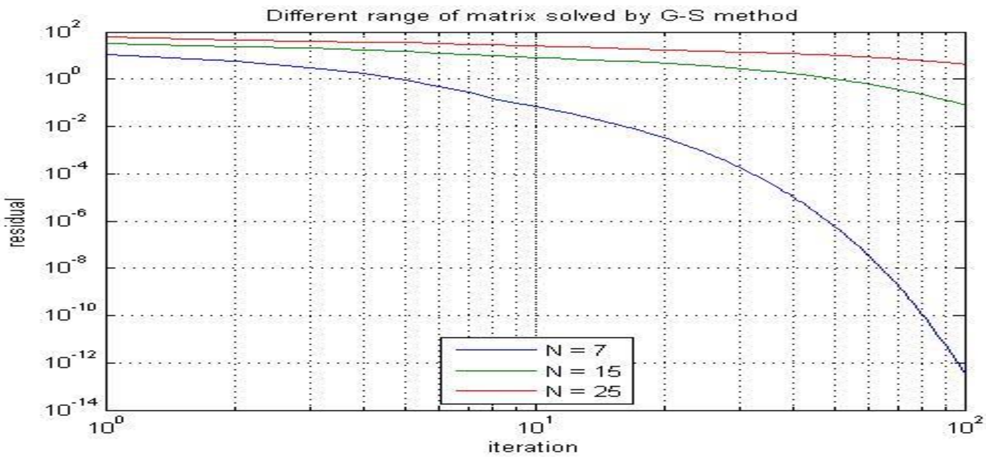

Figure (11): At one end of grid set of positive charges and at other end of grid are set of negative charges and at middle neagaive or positive set of charges

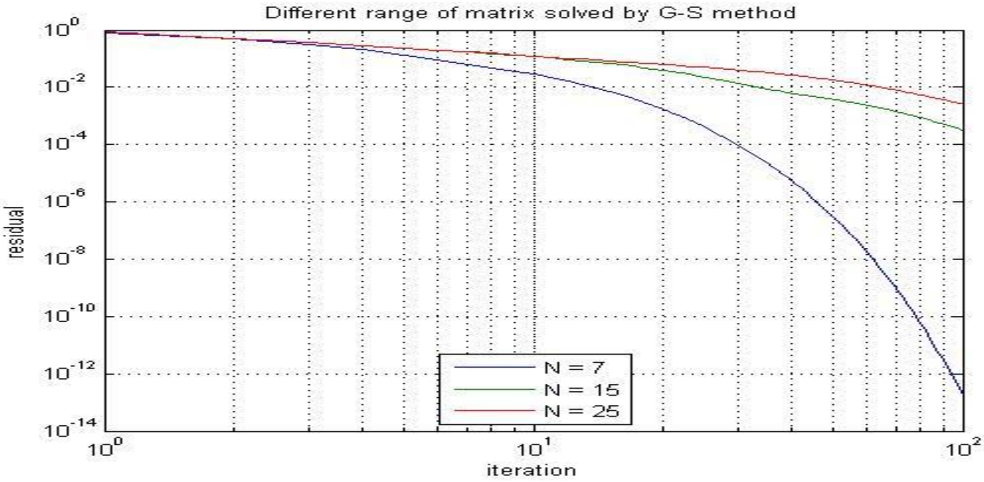

Figure (12): Two charges one positive and other negative located at top and bottom of grid.

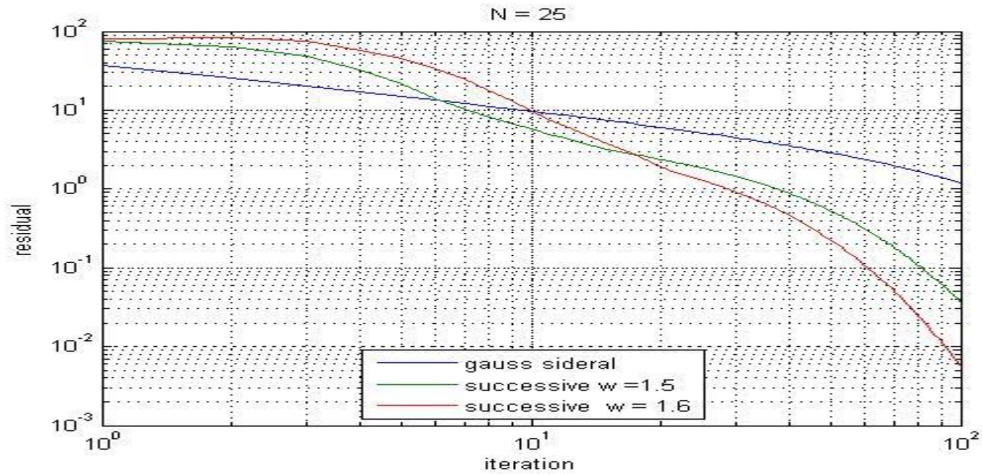

Figure (13): Two positive sets of charges located at grid ends

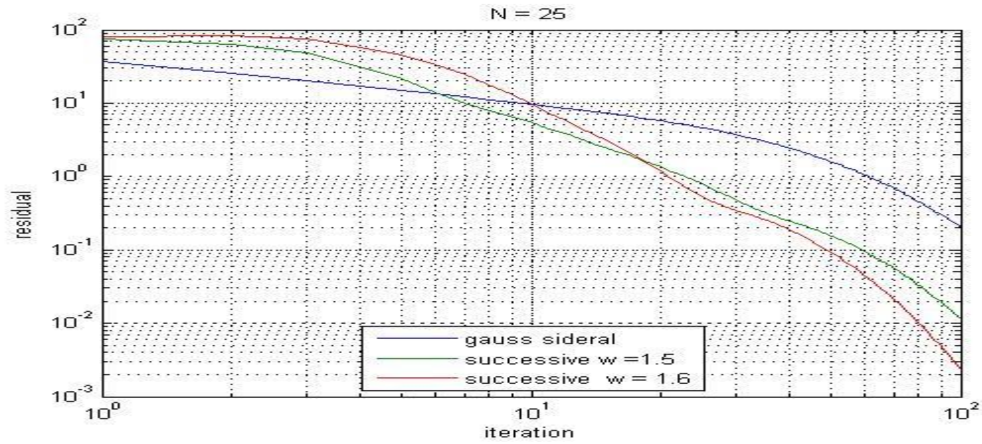

Figure (14): Two sets of different charges located at ends of grid.

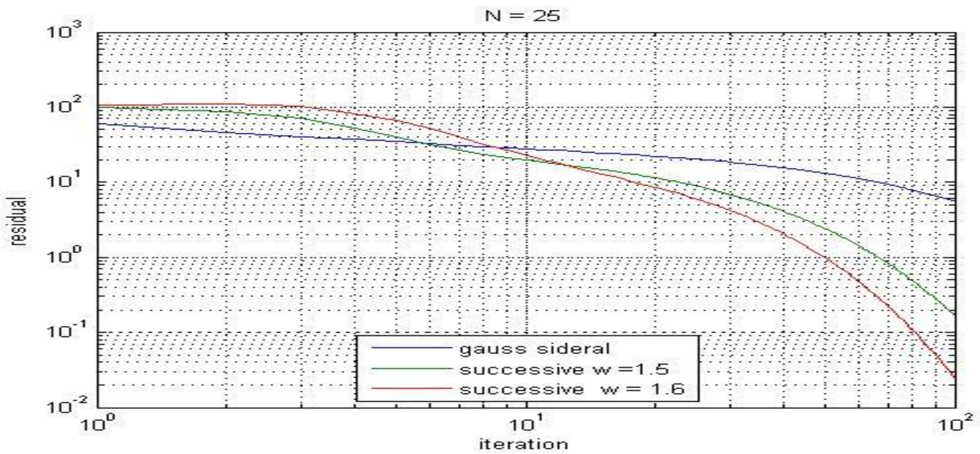

Figure (15): Set of negative charges located at middle of grid and two sets of positive charges located at ends of grid or vice versa two negative sets of charges located at ends of grid and set of positive charges located at middle of grid

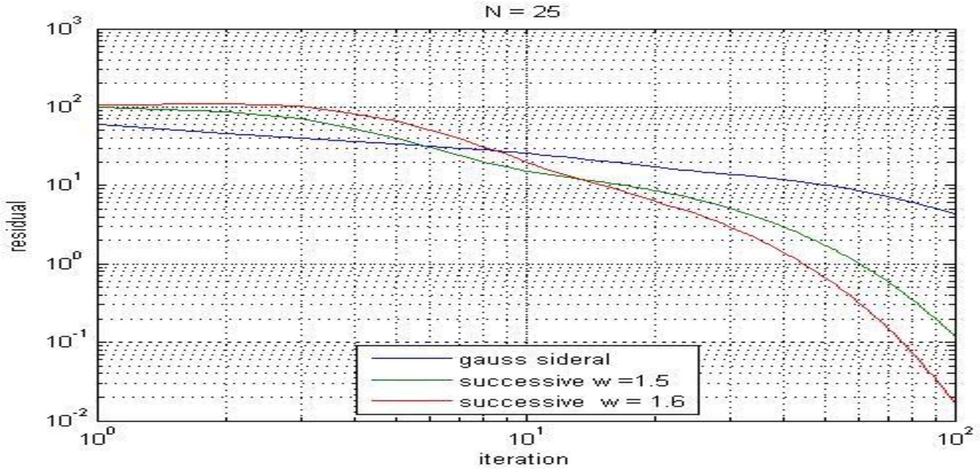

Figure (16): At one end of grid set of positive charges and at other end of grid are set of negative charges and at middle neagaive or positive set of charges

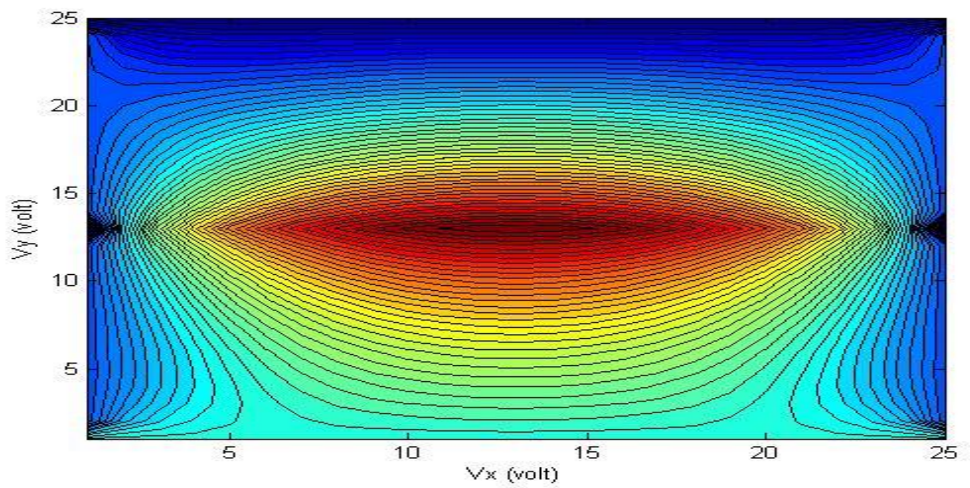

Figure (17): At one end of grid set of positive charges and at other end of grid are set of negative charges and at middle neagaive or positive set of charges