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EFFECT OF PRANDTL NUMBER AND DIAMETER RATIO ON LAMINAR NATURAL CONVECTION FROM ISOTHERMAL HORIZONTAL CYLINDRICAL ANNULI

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ABSTRACT: - A numerical investigation of transient, two-dimensional natural convection in horizontal isothermal cylindrical annuli is performed to investigate the Prandtl number and diameter ratio effect on flow and heat transfer characteristics. The finite difference method is used to solve the governing equations, in which buoyancy is modeled via the Boussinesq approximation. Both vorticity and energy equations are solved using alternating direction implicit (ADI) method and stream function equation by successive over relaxation (SOR) method. Solutions for laminar case are obtained up to Grashof number of 10^5 as well as three different diameter ratios, namely 1.2, 1.5, and 2.0 and the Prandtl number varies from 0.7 to 10 are considered. The computed flow patterns and temperature fields are shown by means of streamlines and isotherms, respectively, and the average heat transfer coefficients are also presented. The numerical results are summarized by Nusselt number vs. Grashof number correlations with the Prandtl number and diameter ratio as a parameter. The results of the parametric study show that the diameter ratio and Grashof number have a profound influence on the temperature and flow field and they are almost independent of a low Prandtl number fluid. The average Nusselt number increases by 25% at large of diameter ratio and Prandtl number. Good agreement with earlier available data is obtained.
***Keywords*:** Natural convection; Cylindrical Annuli; Boussinesq approximation; Flow Pattern

1- INTRODUCTION

Natural convection in an annulus concentric cylinder has been extensively investigated due to the variety of technical applications and practicality such as heat transfer in heat exchanger device, solar collectors, cooling of electrical and electronic components, nuclear reactor, thermal storage system and electrical transmission cables (Alawi et.al. 2014)[(1)].

Number of studies has been conducted to investigate experimentally natural convection in cylindrical annulus [(1-7)]. Kuehn and Goldstein (1976)[(8)] performed an experimental and theoretical–numerical studies on natural convection for air and water in horizontal cylindrical annulus at Rayleigh numbers from $2.1x10^4$ to $9.8x10^5$ at diameter ratio of 2.6. Atayılmaz (2011) [(9)] studied experimentally and numerically natural convection from horizontal concentric cylinders. Two test concentric cylinders are made of copper, and the annulus was filled with water.

The first numerical solution of natural convection between horizontal convection cylinders was obtained by Crawford and Lemlich (1962) [(10)] using Gauss-Seidel iteration approach for Prandtl number of 0.7 and for diameter ratio of 2, 8 and 57. Abbot (1964) [(11)] obtains a solution for diameter ratio close unity using matrix inversion techniques. Mack and Bishop (1968) [(12)] employed a power series expansion valid in the range diameter ratios from 1.15 to 4.15. However, as pointed out by Hodnett (1973) [(13)], if the diameter ratio becomes too large; there is a region in the annulus where convection effects are as important as conduction effects. Powe et. al. (1971)[14] examined the transition from steady to unsteady flow for air with Prandtl number around 0.7.

Charrier-Mojtabi et. al. (1979)[15] solved natural convection problem in horizontal cylindrical annulus using implicit alternating direction scheme and the vorticity-stream function formulation. The transient natural convection between two horizontal isothermal cylinders is formed within the Boussinesq approximation and solved numerically by Tsui and Tremblay (1984) [16]. Natural convection of gases in a horizontal annulus, where the inner cylinder is heated by the application of a constant heat flux and the outer cylinder is isothermally cooled, is studied numerically by Kumar (1988)[17]. A computational analysis of steady laminar natural convection of cold water within a horizontal annulus with constant heat flux on the inner wall and a fixed temperature on the outer surface was done by Ho and Lin (1988)[18]. Results are generated for three values of radius ratio over various ranges of the modified Rayleigh number and density inversion parameter.

Ho et.al. (1989)[19] have been presented a numerical solutions for steady laminar two-dimensional natural convection in concentric and eccentric horizontal cylindrical annuli with constant heat flux on the inner wall and a specified isothermal temperature on the outer wall. Results of the parametric study conducted further reveal that the influence of the Prandtl number is quite weak. A numerical investigation has been performed by Han and Baek (1999) [20], to examine the interaction between radiation and steady laminar natural convection in cylindrical annuli filled with a dry gas. Numerical solutions for the transient natural convection in horizontal isothermal cylindrical annuli within Grashof number based on the inner diameter up to 10^5 in air are presented by Hassan and Al-lateef (2007)[21].

Numerical investigation for three-dimensional natural convection inside horizontal cylindrical annulus have been conducted by Yeh (2002)[22] and Li et.al. (2013)[23]. The effect of nanoparticles on natural convection heat transfer in two-dimensional horizontal annulus is also investigated [24, 25]. Khanafer et.al. (2008)[26] carried out a numerical investigation of natural convection heat transfer within a two-dimensional, horizontal annulus that is partially filled with a fluid-saturated porous medium. The unsteady natural convection flow from a horizontal cylindrical annulus filled with a non-Darcy porous medium was solved by the finite-volume method by Kumari and Nath (2008) [27]. The results show that the annulus completely filled with a porous medium has the best insulating effectiveness.

In the above studies conducted up to now, no studies have been focused on the combined effect of Prandtl number and diameter ratio on flow pattern and heat transfer through horizontal cylindrical annuli. In the present study, the main contribution presents a simulation of the problem as a mathematical model which is solved numerically using finite difference method. The combine effect of Prandtl number, diameter ratio on the natural convection heat transfer characteristics for wide range of Grashof number is taken into consideration.

2. MATHEMATICAL FORMULATION

The sketch of the physical model for convection heat transfer from isothermal horizontal cylindrical annuli is shown in Fig. (1). Consider a fluid layer is enclosed between two concentric cylinders with radii r_i and r_o. Temperatures at the heated inner cylinder surface and the cooled outer one, designated by T_h and T_c, respectively, are to be constant. Flow and temperature fields are assumed to have a symmetric nature with respect to vertical plane (θ=0^o and 180^o) and the region of computation is limited between θ=0^o and 180^o.

The physical system consists of a Newtonian fluid air, in an annulus bounded by two isothermal surfaces. The governing equations for the present study are based on the following assumptions:

- fluid motion and temperature distribution are two-dimensional,
- fluid is viscous and incompressible,
- frictional heating is negligible,

- the difference in temperature between the two isothermal boundaries is small compared with $1/\beta$, and
- fluid properties are constant except for the density variation with temperature.

Assuming the validity of Boussinesq approximation, four governing equations (one continuity and energy, two momentum) in polar coordinate can be written as follows[28,29]:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{1}{r}\frac{\partial v}{\partial \theta}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta} = g\beta(T-T_c)\cos\theta-\frac{1}{\rho}\frac{dp}{dr}+\upsilon(\nabla^2 u-\frac{u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial \theta}) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}) = g\beta(T-T_c)\sin\theta-\frac{1}{\rho}\frac{1}{r}\frac{dp}{dr}+\upsilon(\nabla^2 v-\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial \theta}) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+\frac{v}{r}\frac{\partial T}{\partial \theta}=\alpha(\nabla^2 T) \tag{4}$$

where all constants, variables and operators are dimensional and the coordinates are define as follows:

r: measured from the center of the system,
θ : measured counterclockwise from the downward vertical line,
u : the radial velocity, (positive radially outwards), and
v : the tangential (angular) velocity, (positive in the counterclockwise direction for $0^o \le \theta \le \pi$)

The stream function equation Ψ and vorticity equation Ω are expressed [28]:

$$V=\nabla x \Psi \tag{5}$$

$$\Omega=\nabla x V \tag{6}$$

where both of Ψ and Ω satisfy the following solenoidal condition

$$\nabla.\Psi=0 \tag{7}$$

$$\nabla.\Omega=0 \tag{8}$$

The stream function equation satisfies equation (1) automatically. Then, the relation between Ψ and Ω is presented in the following dimensionless form

$$\Omega=-\nabla^2\Psi \tag{9}$$

Further, the pressure terms can be eliminated by taking the curl of equation (2) and (3) and the dimensionless form of the vorticity transport equation is written as:

$$\frac{\partial \Omega}{\partial \tau}+U\frac{\partial \Omega}{\partial R}+\frac{V}{R}\frac{\partial \Omega}{\partial \theta}=Gr\left(\cos\theta\frac{1}{R}\frac{\partial (R\Theta)}{\partial R}-\sin\theta\frac{1}{R}\frac{\partial \Theta}{\partial \theta}\right)+\nabla^2\Omega \tag{10}$$

In the same manner, the energy equation in the dimensionless form is written as:

$$\frac{\partial \Theta}{\partial \tau}+U\frac{\partial \Theta}{\partial R}+\frac{V}{R}\frac{\partial \Theta}{\partial \theta}=\frac{1}{\mathrm{Pr}}\nabla^2\Theta \tag{11}$$

also,

$$U=\frac{1}{R}\frac{\partial \Psi}{\partial \theta},\quad V=-\frac{\partial \Psi}{\partial R} \tag{12}$$

All above constants, variables and operators are dimensionless. The dimensionless variables that are defined to transfer equations (9-12) into non-dimensional form are as follow:

$$R=\frac{r}{r_i},\ \theta=\theta,\ U=\frac{ur_i}{\upsilon},\ V=\frac{vr_i}{\upsilon},\ \tau=\frac{tv}{r_i^2},\ \Theta=\frac{T-T_c}{T_h-T_c},\ Gr=\frac{g\beta(T_h-T_c)r_i^3}{\upsilon^2},$$

$$\nabla^2=r_i^2\nabla^2 \tag{13}$$

Both of vorticity equation (10) and energy equation (11) are coupled through the buoyancy force. It can be noted that, the vorticity and energy equations are non-linear due to the convective terms. Furthermore, stream function equation (9) is of the elliptic type and both of vorticity and energy equations (10, 11) respectively are of the parabolic type. Equation (9) is couple with equations (10) and (11) through equation (12) which is related the stream function to the velocities.

The solution procedure is to find all of $\Theta(r,\theta,t)$, $\Omega(r,\theta,t)$ and $\Psi(r,\theta,t)$ which satisfy three partial differential equations (9-11) as well as the following initial and boundary conditions. To begin with, the fluid in the annulus is stationary with a uniform temperature:

$$\Omega = \Psi = T = 0 \quad \text{everywhere at} \quad \tau = 0 \tag{14}$$

The following boundary conditions on Ψ and θ are specified for annuli:

$$\Psi = \frac{1}{R}\frac{\partial \Psi}{\partial \theta} = \frac{\partial \Psi}{\partial R} = 0 \text{ on both walls, i.e. } R = 1 \text{ and } R = R_o \tag{15a}$$

$$\theta = 1 \quad \text{at} \quad R = 1 \tag{15b}$$

$$\theta = 0 \quad \text{at} \quad R = R_o \tag{15c}$$

The final form of governing equations are (5, 9-11), which are transformed into the finite difference equations and solved numerically [29]. The relaxation factor is 1.75 for the stream function, and the number of nodal points in the grid was 41, 21 for the R-θ respectively.

3. NUMERICAL SOLUTION

In the present study, a finite difference method is applied to find the numerical solution of two- dimensional transient natural convection from isothermal cylindrical annuli. The solution procedure includes of stream function, temperature fields and velocity distribution in r and θ coordinates.

The alternating direction implicit (ADI) method is used to solve the vorticity and energy equations. The successive over-relaxation method (SOR) is used to obtain the stream function field. The time increment used in the iterative procedure is expressed as [16]:

$$\Delta\tau = \frac{2}{2\left(\frac{1}{(\Delta r)^2} + \frac{1}{(\Delta\theta)^2}\right) + \frac{U}{\Delta r} + \frac{V}{\Delta\theta}} \tag{16}$$

The iterative procedure for the average Nusselt number was repeated until the following convergence criteria is satisfied:

$$\frac{Nu^{n+1} - Nu^{n}}{Nu^{n+1}} \le 10^{-4} \tag{17}$$

Local Nuselt numbers at the inner and outer radius Nu_i and Nu_o are defined as follows:

$$Nu_i = -\ln a\left[R\frac{\partial \Theta}{\partial R}\right]_{r=R_i} \tag{18}$$

$$Nu_o = -\ln a\left[R\frac{\partial \Theta}{\partial R}\right]_{r=R_o} \tag{19}$$

The average Nusselt numbers $\overline{Nu_i}$ and $\overline{Nu_o}$ are the angular average of their local values over the cylinder inner and outer surface, can be carried out using numerical integration by Trapezoidal rule [30]

$$\overline{Nu} = \frac{1}{A}\int_A Nu.dA \tag{20}$$

4. RESULTS AND DISCUSSION

The heat transfer mechanism and fluid flow behavior in a concentric annulus cylinder are strongly depended on the diameter ratio which is defined as a ratio of the diameter of the outer to the inner cylinder. The temperature difference between the heated inner cylinder and cold outer cylinder contributes to the density gradient and circulates the fluid in the annulus.

The next important dimensionless parameters are the Rayleigh and Prandtl numbers, which affect the heat transfer mechanism, the flow pattern and the stability of the transitions of flow in the system[1]. The predicted streamlines and isotherm contours are used to explain the heat transfer within the annulus.

The numerical model was validated with available previous published results. Table (1) shows a comparison between the present study with available previous literature [16] for average Nusselt number at a=2, Pr=0.7 and Gr values of 10 000, 38 000 and 88 000. It can be seen that a good agreement is achieved between the present results and the available numerical results.

In Fig. (2), the effect of diameter ratio on the results of average transient $\overline{Nu_i}$ and $\overline{Nu_o}$ vs. dimensionless time τ are presented. As can be seen that when τ increases, both $\overline{Nu_i}$ and $\overline{Nu_o}$ approach to their steady- state values and should be equal based on a simple energy balance. It can be noted that the dimensionless time increases with increases the diameter ratio and decreases with increases of Grashof number.

The effect of Prandtl number on the results of $\overline{Nu_i}$ and $\overline{Nu_o}$ vs. dimensionless time τ when the diameter ratio varies from 1.2 to 2, are also presented in Figs. (3 and 4). It seen that, at small values of diameter ratio and Prandtl number, there is no significantly change in the convection heat transfer even the Gr value increases. The average Nusselt number corresponding to higher Prandtl number are higher than those for air. When the diameter ratio increases further from 1.2 to 2.0, the convection heat transfer increases extensively, as shown in Fig. (4).

Figures (5) and (6) show the effect of diameter ratio on the flow pattern and temperature fields of annuli for the whole range of present numerical calculations. There is no significant change on flow pattern and temperature fields at lower Grashof and Prandtl numbers. The center of rotation moves towards the top with increasing diameter ratio, but separation is clear at high Grashof number. The flow and heat transfer results can be divided in to several regimes [8]. Near Grashof number of 10^2, the maximum stream function or center of rotation is near 90^o. The flow in the top and bottom portions of the annulus is symmetric about the 90^o position. At different diameter ratio, the isotherms begin to resemble eccentric circles near a Grashof of 10^3. This has been called the 'pseudo-conductive regime [31], since the overall heat transfer remains essentially that of conduction. A transition region exists for Grashof numbers between 10^2 and 10^4. As the Grashof number increases further, the center of rotation moves upwards.

Further increment of both diameter ratio and Grashof number, the temperature distribution becomes distorted, resulting in an increase in average Nusselt number, see Fig. (6). In case of large diameter ratio, the total heat flow will be essentially that from a single horizontal cylinder in an infinite medium. An oscillating laminar flow regime begins near Grashof number of 10^5. As the Grashof number increased further, the flow above the inner cylinder will become turbulent. The rotation center moved near the top as the Prandtl number increased. The velocity vector for natural convection in an annulus as a function of Grashof number when the diameter ratio varies from 1.2 to 2, are presented in Fig. (7). A crescent-shaped eddy dominates for the small diameter ratio and a kidney-shaped flow pattern appears for the large diameter ratio as observed by previous investigators in their flow visualization studies. The velocities are too small at low Grashof number and increases with increases both of Garshof and Prandtl numbers causing the separation of inner and outer cylinder thermal boundary layer.

Finally, the relation between average Nusselt number and Grashof number in terms of Prandtl number and diameter ratio as a parameter are correlated to one-fourth power law with maximum deviation less than 3%, as shown in Fig. (8). It is seen that, at a small diameter ratio, there is a little convective heat transfer even at high value of Grashof number which has been proved by Kuehn and Goldsteins (1976) [32] calculations. Also, it is clear that the maximum increment in amount of convection heat transfer with larger Prandtl number (Gr=10^5) by 25% at a=2.0 respectively compared with corresponding values at low Prandtle number.

5. CONCLUSIONS

A numerical study of transient, two-dimensional natural convection heat transfer problem in isothermal horizontal cylindrical annuli, enclosed in heated inner and cooled outer cylinders is performed. The influence of Prandtl number and diameter ratio on the flow structure and heat transfer is investigated for a wide range of Grashof number. The computed flow patterns and temperature fields are shown by means of streamlines and isotherms, respectively, and the average heat transfer coefficients are also presented. The Prandtl number of the range ($0.7 < Pr < 10$) as well as different diameter ratios (a = 1.2, 1.5 and 2.0) and Grashof number over several orders of magnitude ($Gr = 10^2$, 10^3, 10^4, and 10^5) are considered. The results show that diameter ratio and Grashof number have a profound influence on the flow and heat transfer characteristics and no significant change at low Prandtl number fluid. At large diameter ratio, the total heat flow will be essentially that from a single horizontal cylinder in an infinite medium. The maximum Nusselt number occurs at large of diameter ratio and Grashof number. The average Nusselt number increases by 25% at a =2.0, Pr=10. Good agreement with previous available data is obtained.

NOMENCLATURE

a	diameter ratio, radius ratio, r_o/r_i
c_p	specific heat at constant pressure
g	gravitational acceleration
Gr	Grashof number, $g\,\beta\,(T_h\text{-}T_c)\,r_i^3\,/\upsilon^2$
Δr	mesh interval in r-direction
$\Delta\theta$	mesh interval in θ-direction
k	thermal conductivity
Nu	Nusselt number
Pr	Prandtl number, υ/α
r	radial distance
R	dimensionless radial coordinate, r/r_i
Ra	Rayleigh number, $Ra= Gr.Pr$
T	temperature
u	radial velocity
U	dimensionless radial velocity, ur_i/υ
v	tangential velocity
V	dimensionless tangential velocity, vr_i/υ
Greek symbols	
α	thermal diffusivity,$(k/\rho\,c_p)$
β	thermal expansion coefficient of fluid
Ω	dimensionless voirticity
Θ	dimensionless temperature, $(T\text{-}T_c)/(T_h\text{-}T_c)$
τ	dimensionless time, $t\upsilon/r_i^2$
υ	kinematics viscosity
ρ	fluid density
Ψ	dimensionless stream function
Subscripts	
h,c	hot and cold, respectively
i,o	inner and outer, respectively
Superscripts	

- average

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Table (1): Average Nusselt number results for *a* =2.0, *Pr*=0.7

a	Gr_{Ri}	$\overline{Nu}$ (Present study)	$\overline{Nu}$ (16)
2.0	10 000	1.659	1.64
	38 800	2.42	2.4
	88 000	2.99	3.08

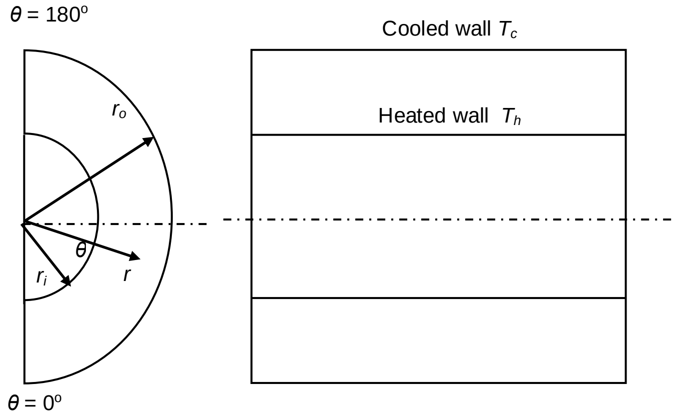
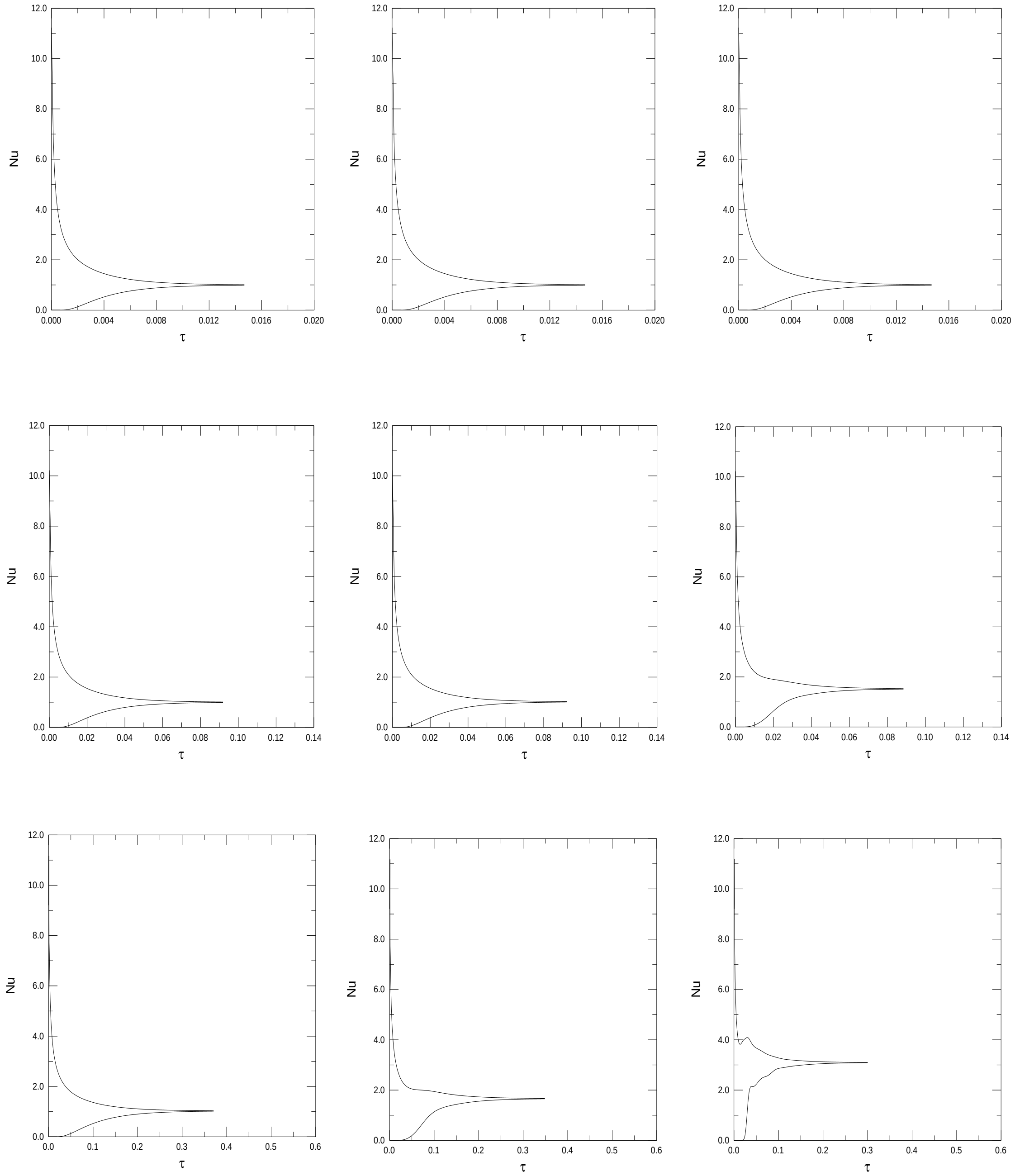


Fig.1: The physical model and coordinate system.

a=1.

a=1.

a=2.



$Gr = 10^3$ $Gr = 10^4$ $Gr = 10^5$

Fig. (2): Average Nusselt number vs. non-dimensional time at different diameter ratios, Pr=0.7

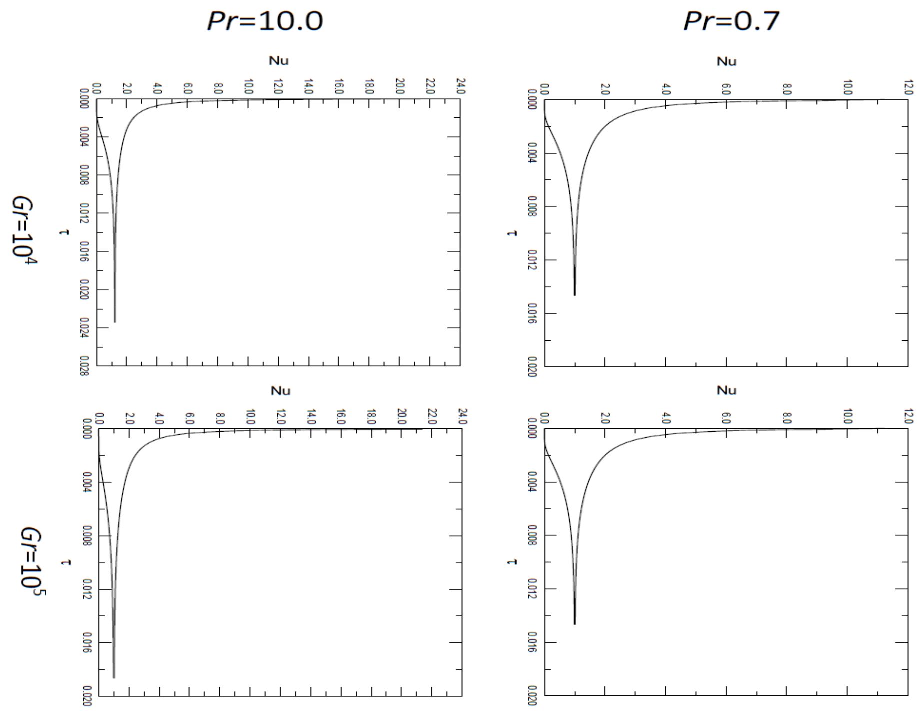


Fig.(3): Average Nusselt number vs. non-dimensional time at different Prandtl numbers, a=1.2

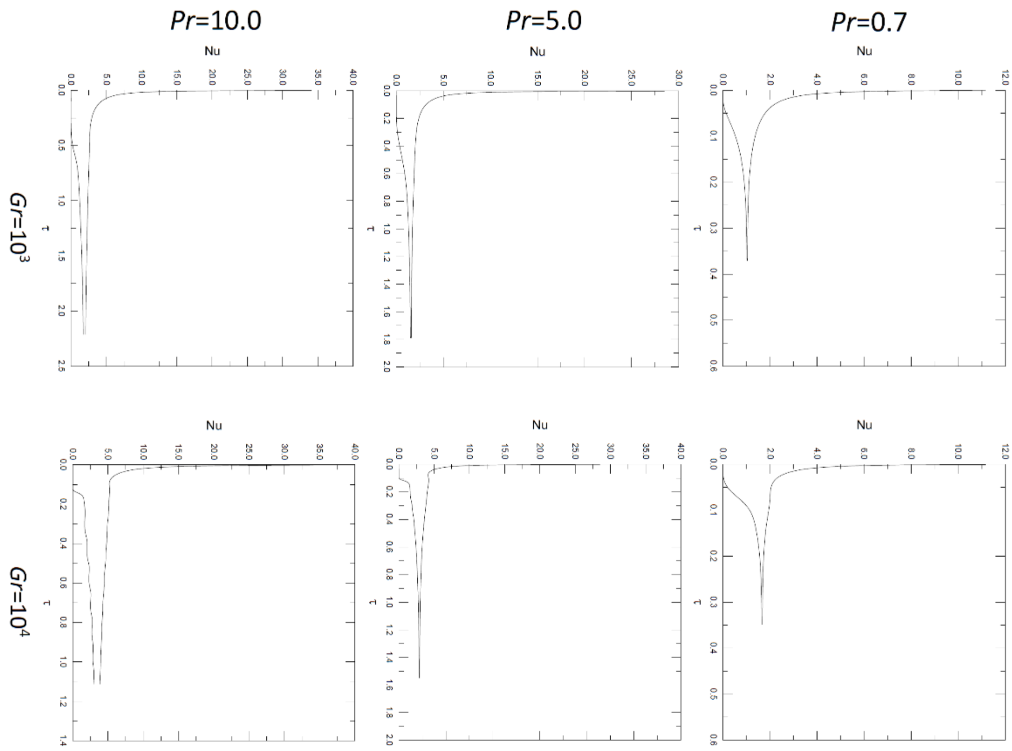


Fig.(4): Average Nusselt number vs. non-dimensional time different Prandtl numbers, a=2.0

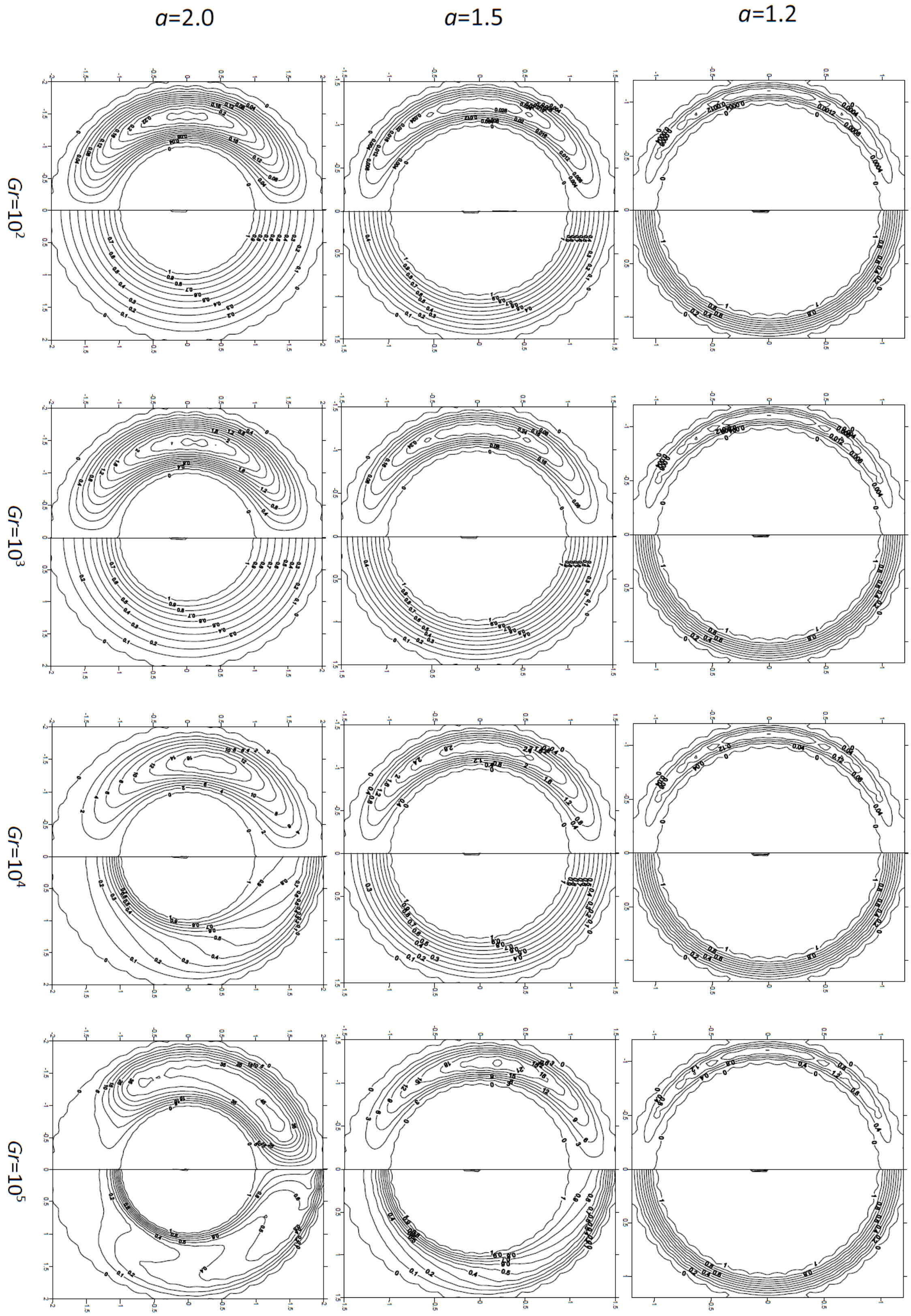


Fig.(5): Streamlines and isotherms at different diameter ratios, Pr=0.7

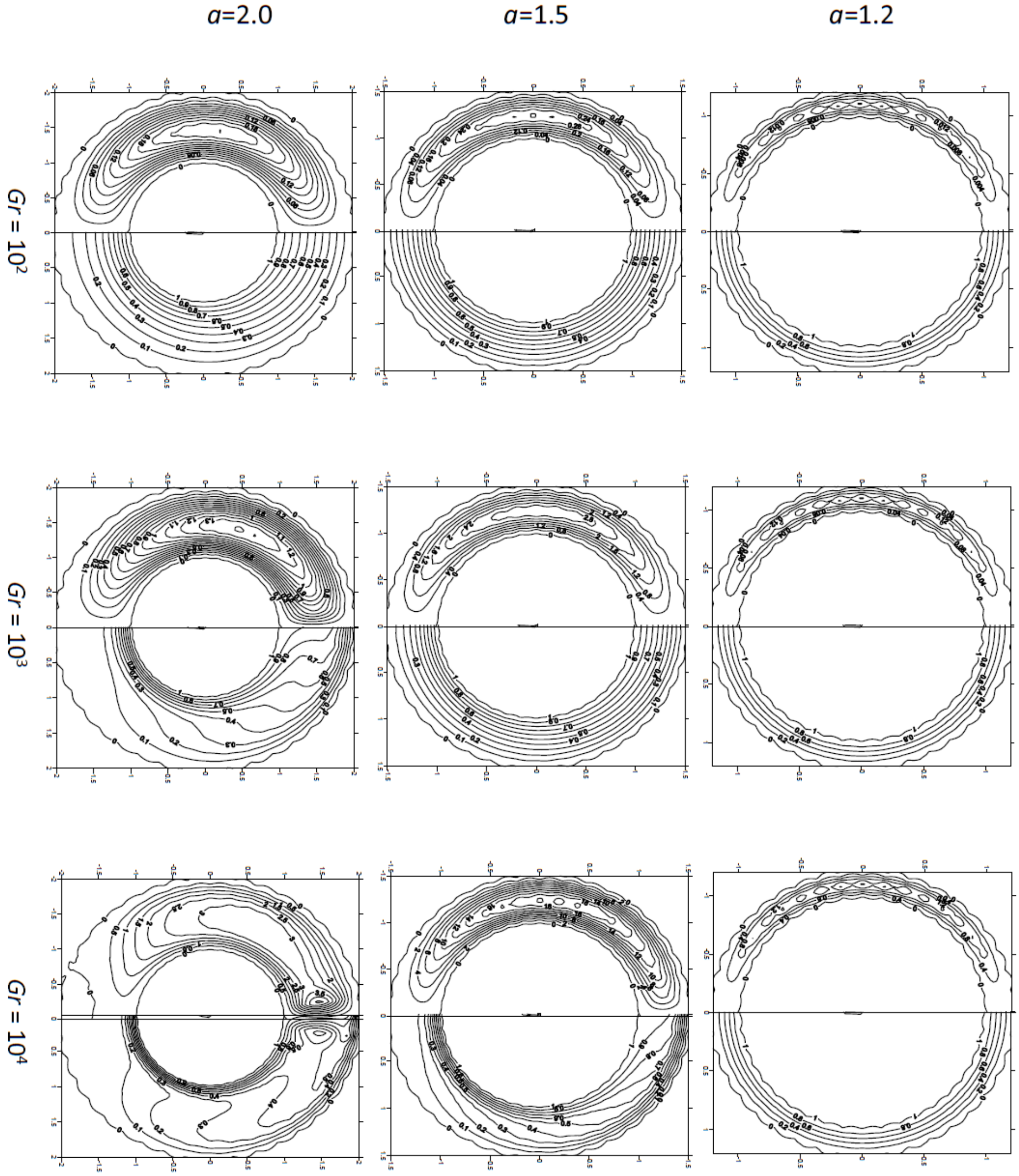


Fig.(6): Streamlines and isotherms at different diameter ratios, Pr=10.0

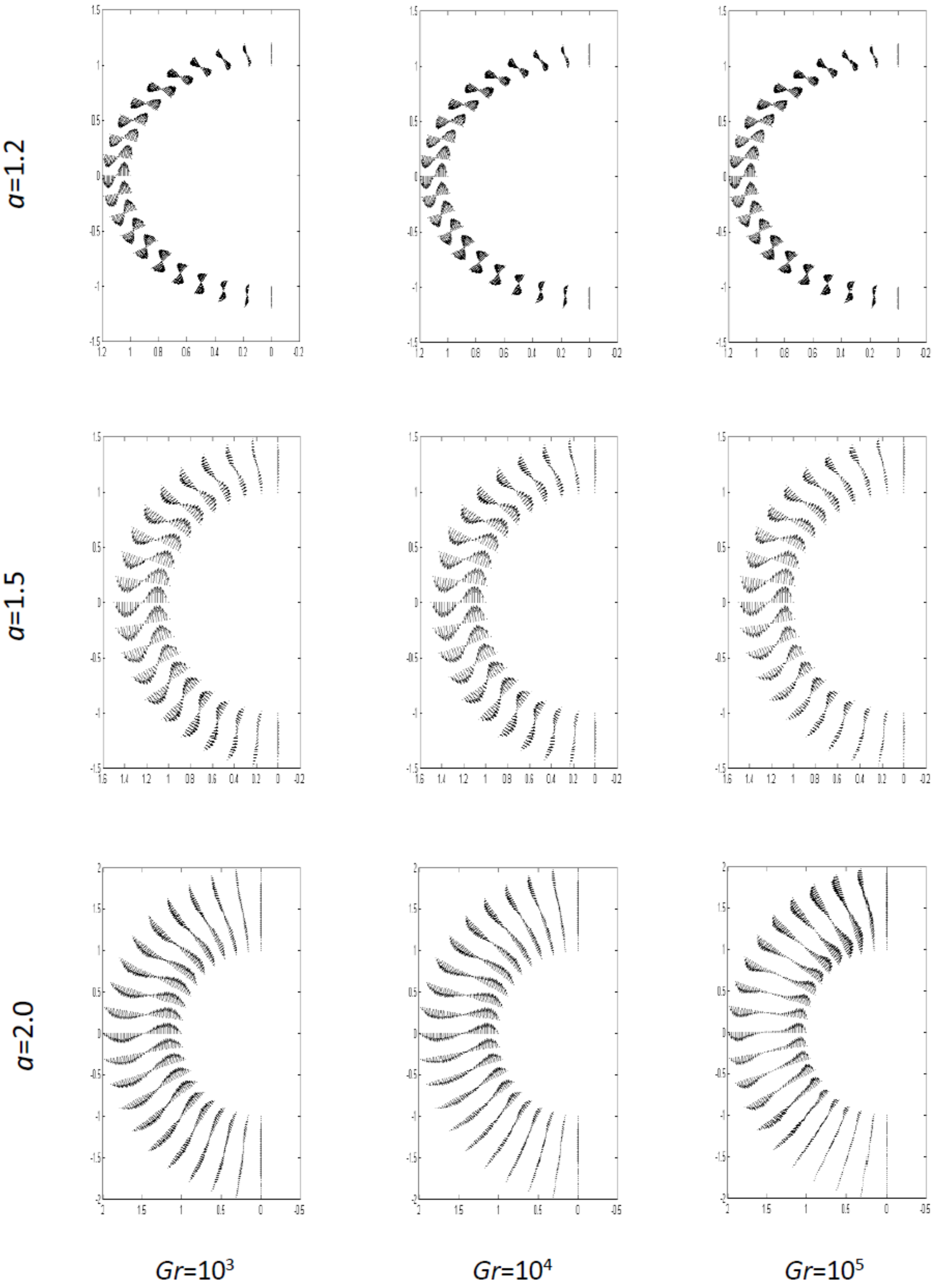


Fig. (7): Velocity vector for natural convection in an annulus at different diameter ratios, *Pr*=0.7

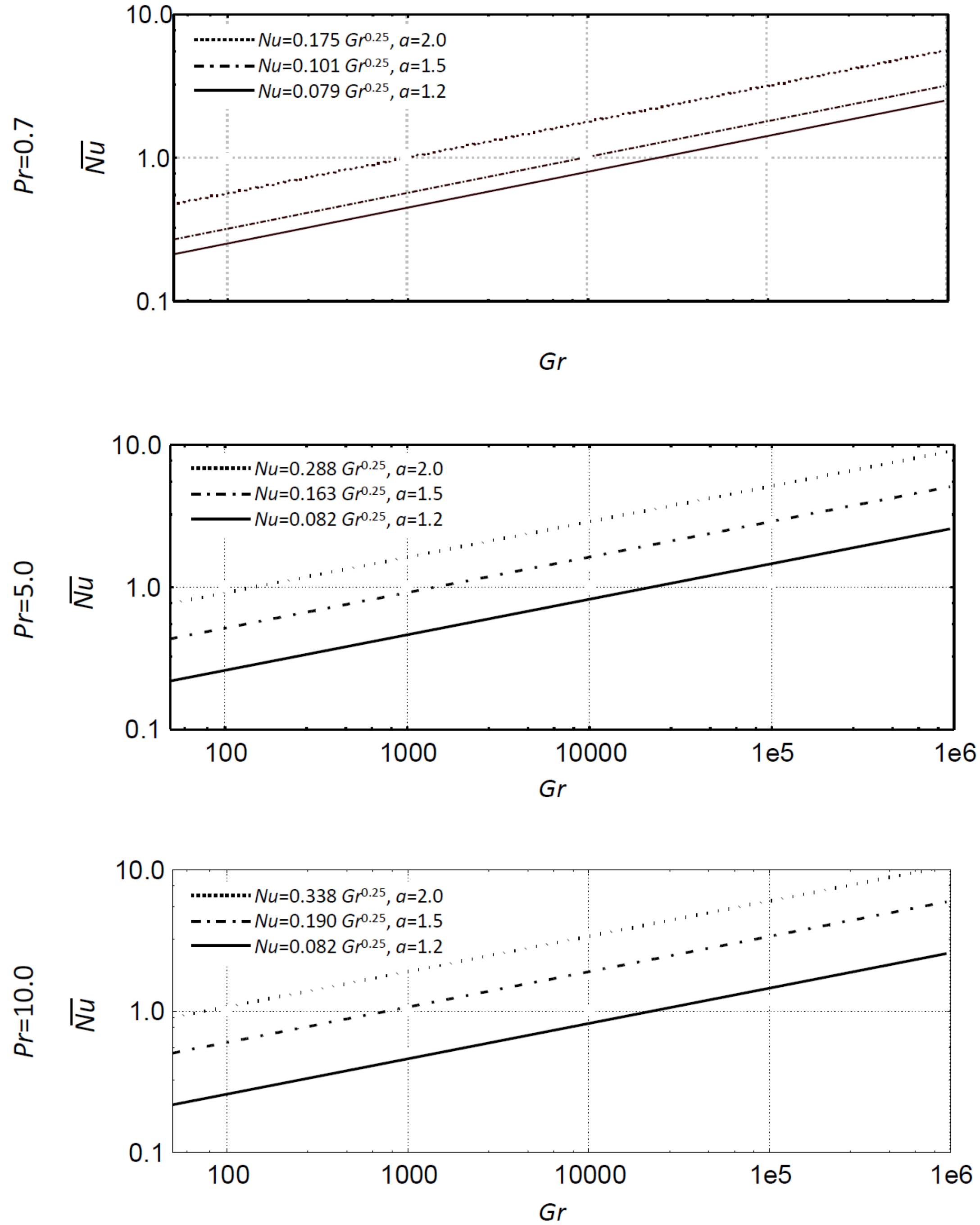


Fig. (8): Nusselt number vs. Grashof number correlations at different Prandtl numbers

تأثير رقم برانتل ونسبة القطر على الحمل الطبيعي الطباقي من عمود حلقي اسطواني ثابت درجة الحرارة بالوضع الافقي

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الخلاصة

يستعرض هذا البحث دراسة عددية لانتقال الحرارة ثنائي الابعاد بالحمل الحراري الطبيعي العابر من عمود حلقي اسطواني ثابت درجة الحرارة بالوضع الافقي، وذلك لبيان تاثير رقم برانتل ونسبة القطر على خصائص الجريان وانتقال الحرارة. تم استخدام طريقة الفروقات العددية المحددة لحل المعادلات الحاكمة. معادلة الدوامية والطاقة تم حلها بأستخدام طريقة الاتجاه الضمني المتناوب بينما دالة الانسياب بطريقة التراخي فوق التعاقب. تم حل المسألة لحالة الجريات الطباقي ولرقم كراشوف لحد 10^5 فضلا عن ثلاث نسب اقطار مختلفة (2.0،1.5،1.2) ولرقم برانتل يتراوح من 0.7 الى 10. أنماط الجريان وتوزيع درجات الحرارة تم تمثيلها بأستعمال كل من خطوط الانسياب ودرجات الحراة الثابتة على التوالي، وكذلك متوسط معاملات انتقال الحرارة. النتائج العددية تم أجمالها بعلاقات تجريبية بين رقم نسلت ورقم كراشوف مع اعتبار تأثير رقم برانتل ونسبة القطر فيها. نتائج الدراسة بينت انه تأثير نسبة القطر ورقم كراشوف الواضح على توزيع الحرارة والجريان مع عدم اعتمادهما في حالة الموائع ذات اللزوجة المنخفضة. تزداد قيمة متوسط رقم نسلت بنسبة 25% عند اعلى قيمة لنسبة القطر ورقم برانتل. تم الحصول على توافق جيد مع الدراسات السابقة